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Transpose of a Matrix

Definition - The matrix obtained from any given matrix A, by interchanging its rows and columns is called the transpose of A and is denoted by A' or AT.

Ex. if $A = \begin{bmatrix} 2 & 4 \\ 6 & 8 \\ 3 & 9 \end{bmatrix}$, then $A' = \begin{bmatrix} 2 & 6 & 3 \\ 4 & 8 & 9 \end{bmatrix}$

It show that if A is a m×n matrix, then A' will be n×m matrix and the (i, j)th element of A in equal to (j, i) th element of A'.

Thus if $A = [a_{ij}]$, then $A' = [a_{ji}]$.

(1) Show that $(A')' = A$

let $A = [a_{ij}]$

$\therefore A' = [a_{ji}]$

$\therefore (A')' = [a_{ij}] = A$

(2) $(CA)' = CA'$, where c in scalar

we know, $cA = c[a_{ij}] = [ca_{ij}]$

$\therefore (CA)' = [ca_{ji}] = c[a_{ji}] = CA'$.

(3) $(A+B)' = A' + B'$, where A and B are conformal for addition.

let $A = [a_{ij}]$ and $B = [b_{ij}]$

$\therefore A+B = [a_{ij}] + [b_{ij}] = [a_{ij} + b_{ij}]$

$\therefore (A+B)' = [a_{ji} + b_{ji}] = [a_{ji}] + [b_{ji}] = A' + B'$.

4. $(AB)' = B'A'$ when A and B are conformal for the product AB.

Proof:- Let $A = [a_{ij}]$ and $B = [b_{jk}]$ be the $m \times n$ and $n \times p$ matrices respectively.

Then $A' = [a_{ji}]$ and $B' = [b_{kj}]$ will be the $n \times m$ and $p \times n$ matrices respectively.

Now $AB = [a_{ij}] \times [b_{jk}] = C$; where $c_{ik} = \sum_{j=1}^{n} a_{ij} b_{jk}$ and it is a $m \times p$ matrix.

$\therefore (AB)' = C'$; where $c_{ki} = \sum_{j=1}^{n} a_{ij} b_{jk} = \left[\sum_{j=1}^{n} b_{jk} a_{ij}\right]$;

where $i = 1, 2, 3, --- m$; $k = 1, 2, 3, --- p$ ——— (1)

Now, the elements in the kth row of B' are the elements of the k^{th} column of B.

They are $b_{1k}, b_{2k}, b_{3k}, --- b_{nk}$

Similarly, the elements of the ith column of A' are the elements of the ith row of A.

They are $a_{i1}, a_{i2}, a_{i3}, -- a_{in}$.

The scalar product of these two sets of elements

$$= \sum_{j=1}^{n} b_{jk} a_{ij}$$

$$\therefore B'A' = \left[\sum_{j=1}^{n} b_{jk} a_{ij}\right]$$

where $i = 1, 2, 3, --- m$; $k = 1, 2, 3, — p$ ——— (2)

Therefore, from (1) and (2), we get

$(AB)' = B'A'$.

This theorem is also called the reversal rule for the transpose of a product.